Tuning the glass-forming ability of metallic glasses through energetic frustration

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(Dated: April 7, 2019)

The design of multi-functional BMGs is limited by the lack of a quantitative understanding of the variables that control the glass-forming ability (GFA) of alloys. Both geometric frustration (e.g. differences in atomic radii) and energetic frustration (e.g. differences in the cohesive energies of the atomic species) contribute to the GFA. We perform molecular dynamics simulations of binary Lennard-Jones mixtures with only energetic frustration. We show that there is little correlation between the heat of mixing and critical cooling rate. Thus, it is clear that a more quantitative theoretical framework is necessary for predicting the GFA.

Bulk metallic glasses (BMGs) are amorphous alloys that possess promising structural, mechanical, and functional properties [1–3]. However, a given BMG may not possess multiple desirable properties, such as high elastic strength and biocompatibility in the case of BMGs used in biomedical applications [4]. Thus, de novo design of BMGs with multi-functional properties is an important goal. A key impediment to progress is that one cannot currently predict the glass-forming ability (GFA) of a given alloy [5]. The most prominent and widely used features for identifying BMGs were suggested by Inoue in 2000 [6]: 1) BMGs are typically multicomponent systems consisting of three or more elements, 2) the size ratios of the three main constituents differ by more than 12%, and 3) the heat of mixing $\Delta H_{\text{mix}}$ among the three main elements is negative. However, there are many examples of metallic glasses that do not obey these rules. First, several binary alloys (such as CuZr) possess GFAs that are comparable to those for multi-component BMGs [7–9]. Also, there are many ternary alloys (e.g. Al, Cu, and V) that have $R_c < 10^6$ K/s, but the diameter ratios among the three elements differ by less than 12% [10]. Further, recent experimental studies have shown that even monoatomic metallic systems can form glasses via rapid cooling [11]. Thus, it is clear that a more quantitative theoretical framework is necessary for predicting the GFA of alloys.

There are two main contributions to the GFA of an alloy, geometric and energetic frustration [12, 13]. Geometric frustration can be achieved in alloys using elements with sufficiently different metallic radii [12, 14, 15], which allows the glass phase to pack more desely, but severely strains the competing crystalline phases. Energetic frustration can be achieved in alloys even with elements of similar sizes, if they possess different cohesive energies and strong interactions between different atomic species. While there have been many computational studies of the variation of $R_c$ with geometric frustration [13, 16, 17], there are few studies that have investigated how energetic frustration in the absence of geometric frustration affects the GFA.

In this Letter, we carry out molecular dynamics simulations of binary Lennard-Jones (LJ) mixtures with atoms of the same size, but different cohesive energies, to understand the critical cooling rate as a function of the degree of energetic frustration. We find several important results: 1) We show that there is little correlation between the GFA and heat of mixing in binary and multi-component metallic glass formers. 2) Instead, we find that there is a particular combination of the difference in the cohesive energies and the pair interactions among different species in binary alloys that yields the best GFA for each composition. 3) We rationalize these findings for binary LJ systems with the best GFA by considering separation fluctuations and chemical ordering [18] among nearest neighbor atoms.

We focus on binary LJ mixtures in three dimensions with vanishing geometric, but tunable energetic frustration. The pairwise interaction potential is:

$$V(r_{ij}) = 4\epsilon_{ij} \left[ \left( \frac{\sigma}{r_{ij}} \right)^{12} - \left( \frac{\sigma}{r_{ij}} \right)^{6} \right],$$

where $\sigma$ is the diameter of atoms $A$ and $B$, $r_{ij}$ is the separation between atoms $i$ and $j$, $\epsilon_{AA}$ and $\epsilon_{BB}$ are the cohesive energies of atoms $A$ and $B$, and $\epsilon_{AB}$ is the interaction energy between $A$ and $B$. The potential is truncated and shifted at $r_{ij} = 2.5\sigma$, and the total potential energy is $V = \sum_{i>j} V(r_{ij})$. We consider $N = N_A + N_B = 2000$ atoms with equal mass $m_A = m_B$ in a cubic box and...
periodic boundary conditions in all directions. Length, energy, pressure, and time scales will be reported in units of σ, εAA, ε/σ^2, and \sqrt{m_Aσ/εAA}.

We first equilibrate each system with a fraction of B atoms, f_B = N_B/N, and combinations of ε_{BB}/ε_{AA} and ε_{AB}/ε_{AA} at high temperature T = 5.0 (using a Nose-Hoover thermostat [19, 20]) and then quench them to low temperature T = 0.01 as a function of cooling rate R. The thermal quenches are performed at fixed pressure P_0 = 10 to avoid cavitation [21]. We find that the particular value of P_0 does not strongly affect the GFA in systems that do not cavitate over the range 10^{-2} < P_0 < 10. (See Supplemental Material.)

FIG. 1. (a) The interaction energy ε_{AB} (normalized by ε_{AA}) from the pairwise heat of mixing ΔH_p plotted versus the cohesive energy ratio ε_{BB}/ε_{AA} for 990 binary alloys involving 45 elements found in metallic glasses [22, 23]. We chose element A, such that ε_{AA} > ε_{BB}. The solid line obeys ε_{AB} = c/\sqrt{ε_{BB}ε_{AA}} with c = 1.09. (b) The ratio of χ_{exp} = ε_{AB}/\sqrt{ε_{AA}ε_{BB}} to the London expression, χ_{London} in Eq. 2, plotted versus χ_{London} for the same data in (a).

To understand the relevant range of parameter space for the cohesive energies, ε_{AA} and ε_{BB}, and interaction energy ε_{AB}, we cataloged these values for 990 binary alloys involving 45 elements that occur in metallic glasses. For this analysis, we chose element A such that ε_{AA} > ε_{BB} and used the pairwise definition of the heat of mixing, ΔH_p(i,j) = (ε_{ii} + ε_{jj})/2 − ε_{ij}, to calculate ε_{AB} [24]. Values for ε_{AA}, ε_{BB}, and ΔH_p were obtained from experimental data [22, 23]. In Fig. 1 (a), we show that binary alloys exist over a narrow range of parameters, 0.5 ≤ ε_{AB}/ε_{AA} ≤ 1.4 and 0.1 ≤ ε_{BB}/ε_{AA} < 1. In contrast, these energetic parameters can exist over a wider range in ionic liquids and molten salts [25, 26]. Albeit with scatter, the experimental data scale as ε_{AB} ∝ \sqrt{ε_{AA}ε_{BB}}, which is similar to the London mixing rule ε_{AB} = χ_{London}\sqrt{ε_{AA}ε_{BB}} [27], where

\[
χ_{London} = \frac{2\sqrt{IAIB}}{IA + IB} \left( \frac{2\sqrt{σ_{AA}σ_{BB}}}{σ_{AA} + σ_{BB}} \right)^6,
\]

(2)

σ_{ij} = (σ_i + σ_j)/2 is the average diameter of atoms i and j, and IA and IB are the ionization energies of atoms A and B. In Fig. 1 (b), we show the ratio of χ_{exp} = ε_{AB}/\sqrt{ε_{AA}ε_{BB}} for the experimental data to χ_{London}. More than 70% of the data obeys the London mixing rule with 1 < χ_{exp}/χ_{London} < 1.25. To more fully understand the effects of energetic frustration on the GFA of binary mixtures, below we independently vary ε_{AB}/ε_{AA} and ε_{BB}/ε_{AA} over a much wider range than in Fig. 1 (a).

To quantify the GFA, we analyze the positional order of the system by measuring the bond orientational order parameter for atom i [28, 29]:

\[
Q_6(i) = \frac{4π}{13} \sum_{m=-6}^{6} \left( \frac{1}{N_i + 1} \left( q_{6m}(i) + \sum_{j=1}^{N_i} q_{6m}(j) \right) \right)^2 1/2
\]

(3)

where q_{6m}(i) = \frac{-1}{N_i} \sum_{j=1}^{N_i} (A_{ij}/A_{tot}) Y_{6m}(θ(r_{ij}), φ(r_{ij})), Y_{6m}(θ(r_{ij}), φ(r_{ij})) is the spherical harmonic of degree 6 and order m, θ is the polar angle and φ is the azimuthal angle of the vector r_{ij} from atom i to j, N_i is the number of Voronoi neighbors of atom i, A_{ij} is the area of the Voronoi cell face separating atoms i and j, and A_{tot} is the total area of all faces of the Voronoi cell for atom i [29].

The bond orientational order can distinguish between disordered systems (Q_6 ≤ 0.3) and systems with crystalline order [e.g. face-centered cubic (FCC) with Q_6 = 0.575, body-centered cubic (BCC) with Q_6 = 0.511, and hexagonal close packed (HCP) Q_6 = 0.485]. In Fig. 2 (a), we show the fraction f of each sample with local FCC, HCP, BCC, and disordered structure (using adaptive common neighbor analysis [30]) in systems with f_B = 0.5 over the full range of cohesive and interaction energies for R = 5 × 10^{-5}. For more than 80% of the systems, the fraction of atoms with FCC or HCP order exceeds 0.99. We show that Q_6(i) has a peak near Q_6 ∼ 0.2. For R > R_c, the systems are disordered and P(Q_6(i)) develops peaks near the values corresponding to FCC and HCP order. The peak near Q_6(i) ≈ 0.535 corresponds to regions of adjacent FCC and HCP order, not to BCC order as shown in Supplemental Material. In Fig. 2 (b), we plot the distribution P(Q_6(i)) for a system with ε_{BB}/ε_{AA} = ε_{AB}/ε_{AA} = 1 and several R. For R > R_c, the systems are disordered and P(Q_6(i)) has a peak near Q_6 ∼ 0.2. For R < R_c, P(Q_6(i)) develops peaks near the values corresponding to FCC and HCP order. The peak near Q_6(i) ≈ 0.535 corresponds to regions of adjacent FCC and HCP order, not to BCC order as shown in Supplemental Material. In Fig. 2 (c), we show that ⟨Q_6⟩ = N^{-1}\sum_{i=1}^{N} Q_6(i) versus R is similar to a logistic function, and R_c can be determined by R_c = (⟨Q_6⟩_0 + ⟨Q_6⟩_∞)/2, where ⟨Q_6⟩_0 and ⟨Q_6⟩_∞ are the values in the limits R → 0 and R → ∞ limits.

What combination of ε_{AA}, ε_{BB}, ε_{AB}, and f_B controls the GFA in alloys? One possibility is the heat of mixing, which can be generalized for multi-component alloys as ΔH_{mix} = 4\sum_{i≠j} f_i f_j ΔH_p(i,j) [24]. In Fig. 3 (a), we show R_c versus ΔH_{mix} (normalized by the average cohesive energy ε) for all binary LJ systems studied. We find little correlation between R_c and ΔH_{mix} in the simula-
FIG. 2. (a) Fraction $f$ of each system (labelled $k = 1, \ldots, 210$) with a given local structure (FCC, HCP, BCC, or disordered) for a slow cooling rate ($R = 5 \times 10^{-2}$) in binary LJ systems with $f_B = 0.5$ over the full range of cohesive and interaction energies. (b) Distribution of the local bond orientational order parameter $P(Q_6(i))$ for systems with $\epsilon_{BB}/\epsilon_{AA} = \epsilon_{AB}/\epsilon_{AA} = 1.0$ over four orders of magnitude in cooling rate $R$. (c) Average bond orientational order parameter $Q_6$ for a slow cooling rate ($R = 10^{-13}$) for nine values of $R$. $R_c = (\langle Q_6 \rangle_0 + \langle Q_6 \rangle_\infty)/2$ is obtained by fitting the data to a logistic function $((\langle Q_6 \rangle - \langle Q_6 \rangle_\infty)/(\langle Q_6 \rangle_0 - \langle Q_6 \rangle_\infty)) = (1 - \tanh[\log_{10}(R/R_c)^{1/k}])/2$, where $\langle Q_6 \rangle_0$ and $\langle Q_6 \rangle_\infty$ are the average bond orientational order in the limits of $R \to 0$ and $\infty$, 0 $< \kappa < 1$ is the stretching factor. The vertical dashed line in (b) indicates the $\langle Q_6 \rangle$ that determines $R_c$ (vertical dashed line in (c)).

FIG. 3. (a) $R_c$ from simulations of binary LJ systems versus the heat of mixing $\Delta H_{\text{mix}}/\tau$, where $\tau$ is the average cohesive energy, for nine values of $f_B$. (b) $R_c$ (in K/s) versus $\Delta H_{\text{mix}}/\tau$ from experiments on 482 metallic glass formers with $n = 2, \ldots, 8$ different atomic species.

We also assembled a database of 482 metallic glass formers with $n = 2, \ldots, 8$ different atomic species (see Supplemental Material). The experimental data is similar to the simulation data; there is no correlation between $R_c$ and $\Delta H_{\text{mix}}$, other than $\Delta H_{\text{mix}} < 0$ for all metallic glasses. Note that the simulations cover a much wider range of $\Delta H_{\text{mix}}/\tau$ than experiments on metallic glasses, but $R_c$ in the simulations corresponds to only rapid cooling, $10^{13}$ to $10^9$ K/s.

In Fig. 4 (a) and (b), we show contour plots of $R_c$ versus $f_B$ for $f_A < 0.1$ and $f_B = 0.1$. We find strong correlations between $R_c$ and $\epsilon_- = (\epsilon_{BB} - \epsilon_{AA})/(\epsilon_{AA} + \epsilon_{AB})$ for binary LJ systems with $f_B = 0.1$ and 0.9. $R_c$ increases with increasing $\epsilon_{AB}$ and increasing $\epsilon_-$ for $f_B = 0.1$, whereas $R_c$ decreases with increasing $\epsilon_{AB}$ and decreasing $\epsilon_-$ for $f_B = 0.9$. For $f_B \gg f_A$ with a majority of $B$ atoms and only a small fraction of $A$ atoms, to have good GFA, the cohesive interaction between $B$ atoms must be small compared to that for $A$ atoms with $\epsilon_{BB} - \epsilon_{AA} < 0$ and the interaction between $A$ and $B$ atoms must be strong with $\tau_{AB} \gg 1$. Similarly, when $f_A \gg f_B$ with a majority of $A$ atoms and only a small fraction of $B$ atoms, to have good GFA, the cohesive interaction between $B$ atoms must be strong (or at least comparable to that between $A$ atoms with $\epsilon_{BB} \approx \epsilon_{AA}$) and the interaction between $A$ and $B$ atoms must be strong with $\tau_{AB} \gg 1$. Note that the $R_c$ contours are symmetric with respect to switching the labels of atoms $A$ and $B$, and thus we only show the region $\epsilon_{BB} - \epsilon_{AA} \leq 0$.

We approximate the $R_c$ contours as straight lines in the $\epsilon_-$ and $\tau_{AB}$ plane for each $f_B$ and plot the slope $k$ versus $f_B$ in Fig. 4 (c). The slope crosses zero near $f_B \approx 0.2$ and reaches a peak value of $k \approx 0.25$ near $f_B \approx 0.8$. As $f_B \to 1$, the system becomes monoatomic with all $B$ atoms, the GFA depends only on $\epsilon_-$, and thus $k \to 0$. As $f_B \to 0$, the system becomes monoatomic with all $A$ atoms, and the GFA is independent of $\epsilon_-$ and $\tau_{AB}$. In this regime, the slope of the contours in the $\epsilon_-$ and $\tau_{AB}$ plane is undefined as indicated by the vertical dashed line in Fig. 4 (c). In Fig. 4 (d), we show that the data for $R_c$ can be collapsed by plotting $\log_{10} R_c$ versus $[\epsilon_- - k(f_B)\tau_{AB}]$. We find that the GFA in binary LJ systems obeys a roughly parabolic form:

$$\log_{10} R_c \approx c_1[\epsilon_- - k(f_B)\tau_{AB}]^2 + \log_{10} R_0, \quad (4)$$

where $c_1 \approx -2$ gives the concavity and $R_0 \approx 10^{-2}$ is the cooling rate in the $\epsilon_- \to 0$ and $\tau_{AB} \to 0$ limits.

There are two striking features about the $R_c$ contours in Fig. 4 (a) and (b). First, $R_c$ increases with increasing $\epsilon_-$ and $\tau_{AB}$ for small $f_B$, indicating that systems with the best GFA possess $\epsilon_{BB} \sim \epsilon_{AA}$ and $\tau_{AB} \gg 1$. To frustrate crystallization for small $f_B$, $\epsilon_{BB}/\epsilon_{AA}$ should be
The domain of experimentally accessible binary alloys is enclosed within the solid pink curve (cf. Fig. 1 (a)). The dashed lines represent linear approximations of the equal-$R_c$ contours. (c) The best fit slope $k$ of the equal-$R_c$ contour lines in the $\epsilon_-$ and $\tau_{AB}$ plane plotted versus $f_B$. For $f_B < 0.05$ (vertical dotted line), $R_c$ is uniform and $k$ is undefined. (d) $\log_{10} R_c$ versus $\epsilon_-$ for all systems studied. The dashed line obeys Eq. 4. Binary LJ systems with $\epsilon_-$ and $\tau_{AB}$ in the experimental range in Fig. 1 (a) are indicated by filled diamonds.

As large as possible, approaching $\epsilon_{BB}/\epsilon_{AA} \to 1$. Similarly, large $\tau_{AB}$ allows the $B$ atoms to act as low mobility defects with root-mean-square (rms) fluctuations $\Delta r_{AB} = \langle r_{AB}^2 \rangle - \langle r_{AB} \rangle^2 < \Delta r_{AA}$ in the low-temperature glass, where $\langle r_{AB} \rangle$ is the average separation between an $A$ atom and a Voronoi-neighbor $B$ atom. (See the Supplemental Material.) Second, $R_c$ increases with decreasing $\epsilon_-$ and increasing $\tau_{AB}$ for large $f_B$. In this case, $\epsilon_{BB} \to 0$ prevents $B$ atoms from clustering. Also, in the large $\tau_{AB}$ limit, the $A$ atoms act as low mobility defects with rms fluctuations $\Delta r_{AB} < \Delta r_{BB}$ in the low-temperature glass.

In the high-temperature liquid, the identities of the nearest (Voronoi) neighbors of atoms $A$ and $B$ are completely random. As the system cools, the identities of the neighboring atoms for each atom type $A$ and $B$ can deviate from random, and such chemical ordering can affect the GFA. For example, we hypothesize that if the competing crystal has large chemical order, the system will possess large GFA since the $A$ and $B$ species must rearrange significantly to form the crystal. To assess this hypothesis, we measured the chemical ordering (i.e. the probability $p_A(N_B)$ for an $A$ atom to have $N_B$ $B$ nearest neighbors when $f_A > f_B$) or the probability $p_B(N_A)$ for a $B$ atom to have $N_A$ $A$ nearest neighbors when $f_B > f_A$) at a slow cooling rate with significant FCC order. In Fig. 5, we show $p_B(N_A)$ for systems with $f_B = 0.9$ and $R_c$ decreasing from (a) to (c). We compare $p_B(N_A)$ to $p_B^{\text{random}}(N_A)$, where we keep the low-temperature structure of the system and randomly assign the labels of the nearest neighbors. We find that the GFA increases with the chemical order, $\sum p_B(N_A) - p_B^{\text{random}}(N_A)$, of the competing crystal. We find similar results for systems with $f_B = 0.1$; the GFA increases with the chemical order, $\sum p_A(N_B) - p_A^{\text{random}}(N_B)$, of the competing crystal. (See Supplemental Material.)

By decoupling geometric and energetic frustration, we have shown that the GFA is not strongly correlated to the heat of mixing, which involves the particular combination of variables, $(\epsilon_{BB} + \epsilon_{AA})/(2\epsilon_{AA} - \epsilon_{BB})$. Instead, we find that the GFA is strongly correlated with $\epsilon_-$ (i.e. the difference in the cohesive energies, not the sum) and $\tau_{AB}$, and we identified the $f_B$-dependent combination of $\epsilon_-$ and $\tau_{AB}$ that controls the GFA for binary LJ systems. We emphasize that it was important to study regions of the $\epsilon_-$ and $\tau_{AB}$ parameter space that were beyond the experimental range of metallic glasses to fully understand the GFA. This work will motivate several important future studies. First, we encourage researchers to experimentally characterize the GFA of binary alloys containing nearly monoatomic elements, yet with large energetic frustration. Second, we are now in a position to understand theoretically the GFA of binary LJ systems with both geometric and energetic frustration. For example, it will be interesting to determine how energetic frustration couples to geometric frustration. For example, should element $A$ with larger cohesive energy possess a larger or smaller metallic radius than element $B$ to yield large GFA?

**ACKNOWLEDGEMENTS**

The authors acknowledge support from NSF MRSEC Grant No. DMR-1119826 (Y.-C.H.) and NSF Grant Nos. CMMI-1462439 (C.O.) and CMMI-1463455 (M.S.). This work was supported by the High Performance Computing facilities operated by, and the staff of, the Yale Center for Research Computing.

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FIG. 5. The probability $P_B(N_A)$ for a $B$ atom to have $N_A$ (Voronoi) nearest neighbors for $f_B = 0.9$ in binary LJ systems cooled at $\epsilon = 5 \times 10^{-5}$ (green wide bars) for combinations of $\epsilon_-$ and $\epsilon_{AB}$ that yield (a) $R_e = 6.6 \times 10^{-4}$ ($\epsilon_{AB} = 0.83$, $\epsilon_- = -0.67$), (b) $5.0 \times 10^{-5}$ ($\epsilon_{AB} = 4.17$, $\epsilon_- = -0.11$), and (c) $< 10^{-6}$ ($\epsilon_{AB} = 5.0$, $\epsilon_- = -0.67$). We also show $P_{B_{\text{random}}}(N_A)$ (thin blue bars) for systems with the same structure, but randomized atomic labels for the nearest neighbors.


(2002).


