Flow and clogging of capillary droplets

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Capillary droplets form due to surface tension when two immiscible fluids are mixed. We describe the motion of gravity-driven capillary droplets flowing through narrow constrictions and obstacle arrays in both simulations and experiments. Our new capillary deformable particle model recapitulates the shape and velocity of single oil droplets in water as they pass through narrow constrictions in microfluidic chambers. Using this experimentally validated model, we simulate the flow and clogging of single capillary droplets in narrow channels and obstacle arrays and find several important results. First, the capillary droplet speed profile is nonmonotonic as the droplet exits the narrow orifice, and we can tune the droplet properties so that the speed overshoots the terminal speed far from the constriction. Second, in obstacle arrays, we find that extremely deformable droplets can wrap around obstacles, which leads to decreased average droplet speed in the continuous flow regime and increased probability for clogging in the regime where permanent clogs form. Third, the wrapping mechanism causes the clogging probability in obstacle arrays to become nonmonotonic with surface tension $\Gamma$. At large $\Gamma$, the droplets are nearly rigid and the clogging probability is large since the droplets can not squeeze through the gaps between obstacles. With decreasing $\Gamma$, the clogging probability decreases as the droplets become more deformable. However, in the small-$\Gamma$ limit the clogging probability increases, since the droplets are extremely deformable and cause clogs as they wrap around the obstacles. The results from these studies are important for developing a predictive understanding of capillary droplet flows through complex and confined geometries.

1 Introduction

Capillary droplets, which consist of immiscible liquid droplets dispersed in another continuous phase, are ubiquitous in food science,[12] the pharmaceutical industry,[34] petroleum products,[50] and pollution remediation.[4] Being able to control how capillary droplets flow and interact with their surroundings is crucial for enhancing food emulsion stability, optimizing drug production and delivery, and improving oil recovery rates, while minimizing environmental risks associated with oil exploration. For example, the flow rate of capillary droplets can be tuned by varying the channel width and by designing obstacle arrays in microfluidic chambers,[8,13] cell sorting instruments,[24,16] and cell encapsulation devices.[17,19] However, we do not yet have a fundamental understanding of the dynamics of capillary droplets flowing through complex, confined geometries as a function of the droplet properties.

Flows of hard, frictional grains through narrow constrictions, such as hoppers, silos, and obstacle arrays, display complex spatiotemporal dynamics.[20,22] Over 50 years ago, Beverloo and co-workers[23] carried out experimental studies of hopper flows for a wide range of dry granular materials and proposed an empirical form for the flow rate $Q$ (in the continuous flow rate regime) versus the orifice width $w$: $Q \sim (w/\sigma_{\text{avg}} - k)^{\beta}$, where $\sigma_{\text{avg}}$ is the mean grain size, the flow arrests for $w/\sigma_{\text{avg}} \lesssim k$, $k \gtrsim 1.5$ for rigid grains, and $\beta$ is the power-law scaling exponent that can be tuned by varying the ratio of the dissipation from particle-particle contacts and from the background fluid.[24] In addition, researchers have found that placing an obstacle $\sim 3\sigma_{\text{avg}}$ above the orifice can reduce clogging in silo flows of hard, frictional grains.[25,31] Experimental and computational studies have also shown that the permeability of an obstacle array to granular flow can be tuned by varying the density, shape, and size of the obstacles.[32,15]

In contrast, for deformable particles, it is possible to find $Q > 0$ with no clogging even for $w/\sigma_{\text{avg}} < 1$. Unlike hard grains, deformable particles can change their shapes to squeeze through
narrow constrictions that are much smaller than their undeformed diameters. However, there have been few studies of flows of deformable particles through narrow constrictions. Many experimental studies have instead focused on hydrogel particle flows in hoppers and silos with \( w/\sigma_{\text{avg}} \sim 2-3 \), which do not give rise to large particle deformation. For example, recent studies have found that hydrogel particles with smaller elastic moduli are less likely to clog and possess flow rates that are much faster than those for hard particles, such as glass beads. These prior results for hydrogel particles do not address how particle flow rates are affected by large particle deformation. Thus, an important open question is: does significant particle shape change (i.e. by more than 50\%) still enhance flow through narrow constrictions and other confined geometries?

The interplay between particle deformability and the geometry of obstacle arrays can give rise to a new physical mechanism, particle wrapping, which can strongly influence the dynamics of capillary droplets as they flow through obstacle arrays. When colliding with an obstacle, the highly deformable droplet can wrap around the obstacle with both ends of the droplet elongating in the flow direction until the energy penalty from the droplet surface tension matches the work done by the fluid driving force. At this point, both ends of the droplet become stationary, and then one end of the droplet moves in the opposite direction of the driving force, causing the particle to unwrap. Particle wrapping can even lead to single-particle clogging in obstacle arrays. In this case, the droplet is in force balance after wrapping around an obstacle, where the fluid forces on the droplet are supported by contact forces on the droplet from neighboring obstacles. Thus, particle wrapping can lead to much longer transit times through obstacle arrays compared to those for hard particles with the same undeformed diameter. Despite its importance, the particle wrapping mechanism has not been studied in detail in droplet flows through obstacle arrays.

A key factor that contributes to the knowledge gap concerning flows of highly deformable particles in confined geometries is the lack of an accurate and efficient computational model for these flows. At submicron length scales, e.g. within microfluidic devices, inertial forces are typically much smaller than capillary forces generated by particle surface tension, which strongly influences the behavior of multi-phase fluid flows. Computational models for multi-phase fluid flows include grid-based Eulerian techniques, such as the volume-of-fluid and Lattice Boltzmann methods, and particle-based Lagrangian techniques, such as the smoothed particle hydrodynamics method. Grid-based methods determine the amount of fluid and gas in each cell and calculate the surface energy by reconstructing the fluid-gas interface. However, this method does not strictly conserve mass, and the surface energy can depend on the interface reconstruction algorithm. For particle-based methods, the multi-phase system is discretized into “particles” with each particle assigned as gas or liquid with a specified mass. Therefore, interface tracking is not needed, and the multi-phase fluid behavior is governed by particle-particle interactions. However, it is challenging to determine a priori the particle-particle interactions that yield the correct contact angles at the multi-phase interfaces. Particle-based methods are also computationally demanding since they simulate the droplet interiors in addition to the interfaces. As a result, most particle-based simulations of multi-phase fluids are limited to short time scales and small systems.

In this article, we describe combined experimental and computational studies that enable the development of new efficient computational methodologies and provide new insights for flows of deformable particles through narrow constrictions and obstacle arrays. In the experimental studies, we measure the speed and shape of capillary droplets as they flow through narrow openings as a function of the orifice width and gravitational driving force. For the computational studies, we develop a new deformable particle (DP) model with surface tension (in two dimensions, 2D) that can be implemented to efficiently model thousands of capillary droplets flowing through complex, confined geometries. Varying surface tension in the experiments by more than a factor of 2 is challenging, and thus we carry out simulations of extremely deformable particles with surface tensions that vary by more than a factor of 105, which accentuates the particle wrapping mechanism. We first validate the DP simulations by comparing the droplet shape dynamics in experiments and simulations for a single droplet flowing through narrow constrictions. We then investigate the important physical parameters that control the flow rate and clogging propensity of single deformable particles moving through obstacle arrays using the experimentally validated DP simulations.

We find several important results. First, when a capillary droplet is pushed by gravity through a narrow constriction, its speed initially decreases as it enters the orifice and deforms. After it reaches a minimum speed, the droplet begins to speed up as it exits the orifice. However, as the speed increases, it can overshoot the droplet’s terminal speed. We find that the amplitude of the overshoot decreases with increasing gravitational driving force (relative to the capillary forces). Second, we show that highly deformable droplets flow more slowly in obstacle arrays compared to droplets with larger surface tension, due to the wrapping mechanism. Thus, for capillary droplets flowing through obstacle arrays, the clogging probability, as characterized by the average distance \( \lambda \) traveled by a droplet before it clogs, is non-monotonic as a function of surface tension. For small values of surface tension, \( \lambda \) first increases with increasing surface tension, reaches a peak that depends on the obstacle spacing, and then decreases with continued increases in surface tension. In contrast, models of soft particles that do not model explicit changes in particle shape cannot capture particle wrapping, and thus cannot accurately model flows of capillary droplets moving through obstacle arrays in the low surface tension limit.

The remainder of the article is organized as follows. In Section 2 we describe the computational methods including descriptions of the soft (semi-rigid) particle model and deformable particle model for droplets and the equations of motion and boundary conditions for discrete element method (DEM) simulations of droplet flows through narrow channels and obstacle arrays. In Section 3 we introduce the experimental setup and method to generate capillary droplets and flows. In Section 4 we describe the main results from the combined experimental and
simulation studies. We first characterize the droplet speed and shape dynamics as a function of the distance \( h \) from the orifice for both oil-in-water experiments and DPM simulations of droplet flows through narrow channels. We calibrate the results for the droplet speed and shape from the DP simulations to those from the experiments. We then quantify the nonmonotonic behavior in the droplet speed profile and the clogging probability of single droplets flowing through obstacle arrays as a function of the droplet surface tension. In Section 5 we provide conclusions and propose future research directions, including studies of multi-droplet flows through obstacle arrays with droplet breakup and coalescence that can predict the steady-state droplet size distribution. We also include five Appendices to supplement the discussion in the main text. In Appendix A, we present experimental data for the near-wall droplet speed profile. In Appendix B, we investigate how the results of the DP simulations depend on the number of vertices \( N_i \) in each deformable particle and the perimeter elasticity \( K_i \). In Appendix C, we provide additional details concerning the definition of a clogging event and the clogging probability for droplets flowing through obstacle arrays. In Appendix D, we identify the surface tension regime for which the simulations can achieve continuous flows in obstacle arrays. In Appendix E, we quantify the dependence of the steady-state speed of the droplets on the near-wall drag coefficient.

2 Simulation Methods

In this section, we describe the methods for simulating 2D gravity-driven flows of a single droplet through narrow channels and obstacle arrays. We first illustrate the geometry of the narrow channels and obstacle arrays. We then introduce two models for describing the droplet shape and interactions with the walls: 1) the soft particle (SP) model, which tracks the centers of mass of the droplets and enforces excluded volume interactions using pairwise purely repulsive forces that increase with the overlap between a droplet and the walls and 2) the deformable particle (DP) model that treats each droplet as a polygon with \( N_i \) vertices, employs a shape-energy function to penalize changes in droplet area and changes in the separations between vertices, and enforces excluded volume interactions using pairwise purely repulsive forces between the droplet vertices and walls. For both models, we define the forces on the droplets that arise from the shape-energy function, droplet-wall interactions, and dissipative forces from the background fluid, and provide the equations of motion for the droplet trajectories. Finally, we describe the method for initializing the droplet positions and velocities in narrow channels and obstacle arrays.

2.1 Geometry for flows through narrow channels and obstacle arrays

The narrow channels in the simulations are constructed using two infinitely long wedges, one on the top and one on the bottom that form an orifice between them with width \( w < \sigma \), where \( \sigma \) is the diameter of the undeformed droplet. (See Fig. 1(a).) The obstacle arrays are located within \( L \times L \) simulation boxes with periodic boundary conditions and \( L = 500 \sigma \). The circular obstacles with diameter \( \sigma_{ob} \) are placed inside the simulation box using a Poisson sampling procedure with separations \( w > w_{ob} \). (See Fig. 1(b).)

2.2 Deformable particle model

When droplets squeeze through narrow constrictions, they can experience dramatic shape changes. To efficiently describe large changes in shape that occur during gravity-driven droplet flows, we employ the recently developed deformable particle (DP) model. In 2D, the droplets are modeled as deformable polygons with \( N_i \) vertices. The shape-energy function \( U_s \) for each deformable particle includes three terms:

\[
U_s = \frac{k_u}{2} (a - a_0)^2 + \frac{k_l N_i}{2} \left( \sum_{i=1}^{N_i} (l_i - l_0)^2 \right) + U_{\gamma}.
\]  

The first term imposes a harmonic energy penalty for changes in the droplet area \( a \) from the preferred value \( a_0 \) and \( k_u \) controls the fluctuations in droplet area. The second term imposes a harmonic energy penalty for deviations in the separations \( l_i \) between adjacent vertices \( i \) and \( i+1 \) from the equilibrium length \( l_0 \) and \( k_l \) controls fluctuations in the separations between adjacent vertices. This term ensures that vertices are distributed evenly on the droplet surface, preventing them from aggregating in the direction of gravity. The factor of \( N_i \) in the numerator of the second term of Eq. \( 1 \) makes \( U_s \) independent of \( N_i \). We characterize the shape of the droplet using the dimensionless shape parameter: \( \omega = (\sum_{i=1}^{N_i} l_i)^2 / (4 \pi a) \). The third term gives the energy arising from line tension:

\[
U_{\gamma} = \gamma_{owl} l_{ow} + \gamma_{ow} (L - l_{ow}) + \gamma_{lowl},
\]  

where \( \gamma_{owl}, \gamma_{ow}, \) and \( \gamma_{lowl} \) are the solid-oil, solid-water, and oil-water line tension coefficients. (See Fig. 1(c) for a schematic of the solid wall-oil droplet-water interface in 2D.) \( l_{ow} \) and \( l_{ow} \) are the contact lengths at the oil-solid and oil-water interfaces, respectively. \( L - l_{owl} \) is the length of the solid wall that is not in contact with the oil droplet. In the oil-in-water experiments described here, thin layers of water exist between the oil droplets and chamber walls, thus effectively setting \( \gamma = \gamma_{owl} = \gamma_{ow} = \gamma_{lowl} \).

We assume that there are purely repulsive forces between the droplets and hopper walls or obstacles, which are modeled using the following pairwise interaction potential:

\[
U_w = \sum_{i=1}^{N_i} \frac{\varepsilon_w}{2} \left( 1 - \frac{2d_i}{\delta} \right)^2 \Theta \left( 1 - \frac{2d_i}{\delta} \right),
\]  

where \( \varepsilon_w \) sets the strength of the repulsive interactions, \( d_i \) is the minimum distance between the center of vertex \( i \) of the droplet and the wall or obstacle surface, and \( \delta \) is the vertex diameter. The Heaviside step function \( \Theta(\cdot) \) ensures that the pair forces are non-zero only when vertex \( i \) overlaps a chamber wall or obstacle.

The total potential energy \( U \) for the droplet is the sum of the shape-energy function \( U_s \), gravitational potential energy \( U_g \), and particle-wall interactions \( U_w \),

\[
U = U_s + U_g + U_w,
\]  

where \( U_g = -Mgh, \) \( h \) is the height of the center of mass of the
droplet, \( g \) is the gravitational acceleration, \( M = \rho a_0 \) is the mass of the droplet with areal mass density \( \rho \) and area \( a_0 = \pi \sigma^2/4 \).

We also consider dissipative forces from the background fluid acting on each droplet vertex \( i \):

\[
\vec{F}_i^c = -\frac{\zeta_i}{N_v} \vec{v}_i,
\]

where \( \vec{v}_i \) is the velocity of vertex \( i \) and \( \zeta_i \) is the viscous drag coefficient:

\[
\zeta_i = b_\infty \left( 1 + \frac{b_0}{b_\infty} \frac{\sigma}{d_i} \right)^{\alpha_i}.
\]

In Eq. 5, \( b_\infty \) is the drag coefficient when the droplet is far from the walls, \( d_i \) is the minimum distance between the center of vertex \( i \) and the chamber wall (or obstacle), and \( b_0 \) is the near-wall drag coefficient. The exponent \( \alpha = 4/3 \) is calibrated to the experiments on single droplet flows in narrow channels described in Appendix A.

Thus, for the DP model, the equations of motion for the position \( \vec{r}_i \) of droplet vertex \( i \) is

\[
M_i \frac{\partial^2 \vec{r}_i}{\partial t^2} = -\vec{\nabla}_r U + \vec{F}_i^c,
\]

where \( M_i = M/N \) is the mass of vertex \( i \).

From Eqs. \( 13 \) we introduce the dimensionless line tension \( \Gamma = \gamma/(g \rho \sigma^2) \) and three additional dimensionless energy scales for the DP model in a gravitational field in 2D: \( K_a = k_a \sigma/(g \rho) \), \( K_l = k_l \sigma/(g \rho \sigma) \), and \( E_w = \varepsilon_w/(g \rho \sigma^3) \). In these studies, we vary \( \Gamma \) in the range \( 10^{-2} < \Gamma < 10 \). We set \( K_a > 10^4 \) so that the fluctuations in the droplet areas are small, \( K_l / \Gamma < 10^{-3} \) so that the line tension energy dominates the perimeter elasticity (see Appendix B), and \( E_w = 10^4 \) to minimize the vertex-wall (and vertex-obstacle) overlaps. The time \( t \) is measured in units of \( t_0 = \sqrt{\sigma/g} \) and the equations of motion are integrated using a modified velocity-Verlet algorithm with time step \( \Delta t = 10^{-3} t_0 \).

### 2.3 Soft particle model

We will also describe gravity-driven single-droplet flows through obstacle arrays using the soft particle (SP) model, and compare the results from the SP model to those of the DP model to better understand the influence of droplet shape change on the flows.

In general, there are three contributions to the total potential energy: 1) the shape-energy function \( U_s \), 2) the gravitational potential energy \( U_g \), and 3) the droplet-obstacle interaction energy \( U_w \). For the SP model, \( U_s = 0 \) and excluded volume interactions between the droplet and obstacles are generated by allowing overlaps between them. The pairwise droplet-obstacle interaction energy is

\[
U_w = \frac{\varepsilon_w}{2} \left( 1 - \frac{2d}{\sigma} \right) \theta \left( 1 - \frac{2d}{\sigma} \right),
\]

where \( \sigma \) is the SP droplet diameter, \( d = r - \sigma_{ob}/2 \) is the distance between the center of the droplet and the obstacle surface, \( \sigma_{ob} \) is the obstacle diameter, \( r \) is the separation between the center of the droplet and the center of the closest obstacle, and \( \varepsilon_w \) is the characteristic energy of the repulsive interactions. The Heaviside step function \( \theta(\cdot) \) ensures that the pair forces are non-zero only when the droplet overlaps with an obstacle. Thus, the total potential energy of an SP droplet in an obstacle array is

\[
U = U_g + U_w.
\]

For the SP model, we consider following form for the viscous drag forces on droplets moving in a background viscous fluid:

\[
\vec{F}_s^c = -\zeta_{sp} \vec{v},
\]

where \( \zeta_{sp} \) is the drag coefficient and \( \vec{v} \) is the velocity of the SP droplet. We note that, unlike Eq. \( 5 \) for the DP model, we set \( \zeta_{sp} = b_\infty \). In contrast to the DP model, the SP model considers droplet deformation by allowing overlaps between the droplets and obstacles, which makes it difficult to describe distance-dependent viscous drag. For the SP model, the equations of motion for the
Fig. 2 Series of images of a single droplet flowing through a narrow orifice with width \( w = 0.4\sigma \) from (a) oil-in-water experiments with undeformed droplet diameter \( \sigma \approx 3.5\) mm and tilt angle \( \theta \approx 28^\circ \) and (b) a DP simulation in 2D at dimensionless line tension \( \Gamma = \Gamma^* \) and near-wall drag coefficient \( b_0 = b^* \). The scale bar indicates 1 mm. Below panel (a), we provide the distances of the droplet center of mass to the orifice \( h \) at which the images are captured. We find that \( \Gamma^* \approx 0.16 \) and \( b^*/b_\infty \approx 0.064 \) minimize the deviation in the droplet’s center of mass speed \( \Delta v \) between the DP simulations and experiments. These best-fit values give \( \Delta_h \approx 0.09 \) and \( \Delta_x \approx 0.01 \). In panel (a), we overlay the shape of the droplet from the DP simulations at \( h = 0 \) (red dashed line) onto the corresponding droplet image for the experiments (black solid line). Droplet (c) shape parameter \( A \) and (d) center of mass speed in the driving direction \( v_g \) plotted as a function of \( h/\sigma \) for both experiments (dashed lines) and DP simulations (solid lines). We estimate the dimensionless surface tension in the experiments to be \( \Gamma_{\text{exp}} \approx 0.57 \).

droplet position \( \vec{r} \) are

\[
M \frac{d^2 \vec{r}}{dt^2} = -\nabla U + \vec{F}_\zeta. \tag{11}
\]

We integrate Eq. 11 using a modified velocity-Verlet integration scheme with time step \( \Delta t = 10^{-3}t_0 \). For the SP model, an important dimensionless energy scale is the ratio of the excluded volume interactions to the gravitational potential energy, \( E_w = E_w/(g\rho\sigma^3) \). Here, we vary \( E_w \) over the range \( 10^{-1} < E_w < 10^2 \).

2.4 Simulation initialization

For the DP model, we initialize the droplets in narrow channels and obstacle arrays in the shape of regular polygons with edge lengths equal to their equilibrium values \( l_0 = \sqrt{4\pi N_0 \tan(\pi/N_0)}/N_0 \). For droplet flows in narrow channels, we initially place the droplet far from the orifice, typically at \( h = -10\sigma \), so that the droplet reaches terminal speed before touching the channel walls. For droplets in obstacle arrays, we initially place the droplet in the center of the simulation box. However, this placement typically causes overlaps between the droplet and nearby obstacles. Therefore, after the initial placement of the droplet, we carry out energy minimization (in the absence of gravity) to eliminate overlaps between the droplet and obstacles. We then integrate the equations of motion with gravity and the droplet begins to flow through the obstacle array until it reaches steady state or clogs. (See Sec. 4.2.)

3 Experimental Methods

Below, we will compare the SP and DP simulation results for gravity-driven droplet flows in narrow channels and obstacle ar-
Oil-in-water droplets are prepared via direct injection of the oil into the chamber. We use octane with density $\rho_o \approx 0.703$ g/ml suspended in water with density $\rho_w \approx 0.997$ g/ml. Despite the absence of other oil droplets, we find that it is still necessary to use a 0.5% Tween 20 nonionic detergent solution to prevent sticking of the oil droplets to the microscope slide surfaces.

We image the droplets using a 1.6× lens (0.05 Numerical Aperture) attached to a Leica DM4500 B inverted microscope. Movies are acquired by attaching a 0.35× C-mount to a ThorLabs DCC1545M camera, and recording at a frame rate commensurate with the droplet speed. The microscope is tilted slightly from the horizontal to cause gravitationally driven motion of the droplet. In particular, the tilt angle of the microscope is changed by placing a spacer underneath the front edge of the microscope, and the angle is measured relative to the ground via a Wixley digital angle gauge.

The droplet motion is affected by viscous drag from the water phase (viscosity $\eta_w \approx 1$ mPa-s) and viscous drag between the oil and glass plates ($\eta_o \approx 100$ mPa-s). The droplets try to remain round due to surface tension ($\gamma_{ow} \approx 10$ mJ/m$^2$, as measured by the “Dropometer” from Droplet Labs). However, under gravity, the droplets deform when pressed against other droplets or the sample chamber walls.

We can define a quasi-2D dimensionless surface tension for the experimental studies:

$$\Gamma_{exp} = \frac{\gamma_{exp}}{\sigma \eta_0 \sigma^2}. \quad (12)$$
Here, $\gamma_{\text{exp}}$ is the oil-water interfacial surface tension, $g_{\text{exp}} = \sin \theta g (\rho_w - \rho_o)/\rho_o$ is the acceleration imposed by oil-in-water buoyancy, $g \sim 9.8 \text{ m/s}^2$ is the gravitational constant, $\sigma$ is the average undeformed diameter of the droplets, and $\theta$ is the microscope tilt angle.

4 Results

In this section, we describe the results from the numerical simulations of gravitationally-driven droplet flows through narrow channels and obstacle arrays. We first present a method to calibrate the DP simulations to experimental results for droplet flows through narrow channels. We then show results for the droplet speed in the direction of the driving $V_g$ as a function of distance from the orifice. We find that the droplet speed profile is nonmonotonic and can even possess an overshoot, such that the droplet speed exceeds the terminal speed $V_t$ as it exits the orifice. Using the DP and SP models, we also investigate gravity-driven droplet flows through obstacle arrays. For the DP simulations, we find non-monotonic behavior for the clogging probability as a function of the dimensionless line tension $\Gamma$. We show that droplets can clog at small $\Gamma$ when they are highly deformable and can wrap around the obstacles. We emphasize that the SP model does not include the wrapping mechanism since wrapping requires significant changes in the droplet shape. As shown in many previous studies of flows of nearly hard particles in confined geometries, the droplets can also clog when they are nearly rigid at large $\Gamma$. At intermediate $\Gamma$, we find a regime of continuous droplet flows through the obstacle arrays. We observe qualitatively similar results over a wide range of obstacle densities.

4.1 Calibration of DP model using experiments of a single droplet flowing through narrow channels

We first carried out both experiments and DP simulations of a single droplet flowing through narrow channels under the influence of gravity. To enable a comparison between the experimental and simulation results, we define the root-mean-square deviations in the droplet shape parameter $\Delta_{\text{sh}}$ and speed $\Delta_{V}$:

$$\Delta_{\text{sh}} = \sqrt{\frac{\sum_{i=1}^{N_f} (\alpha_{\text{exp},i} - \alpha_{\text{sim},i})^2}{N_f}} \quad (13)$$

and

$$\Delta_{V} = \sqrt{\frac{\sum_{i=1}^{N_f} (V_{\text{exp},i} - V_{\text{sim},i})/V_{\text{sim},i})^2}{N_f}} \quad (14)$$

where $\alpha_{\text{exp},i}$ is the droplet shape parameter in the $i$th frame in the experimental video, and $\alpha_{\text{sim},i}$ is the droplet shape parameter in the DP simulations when the droplet center of mass is at the same location as it is in the experiments at frame $i$. Similarly, $V_{\text{exp},i}$ and $V_{\text{sim},i}$ are the droplet speed in the direction of gravity at the $i$th frame in the experiments and simulations. The summations in Eqs. (13) and (14) are over all $N_f$ frames in the experimental videos. Despite using a small time step in the DP simulations, we might not find a frame in the simulations where the droplet has the same position as that in the experiments. In this case, we use linear interpolation between frames to generate $\alpha_{\text{sim},i}$ and $V_{\text{sim},i}$.

We seek to minimize $\Delta_{\text{sh}}$ and $\Delta_{V}$ by tuning the dimensionless line tension $\Gamma$ and near-wall drag coefficient $b_0/b_o$ in the DP simulations. In practice, $\Delta_{\text{sh}}$ is more than an order of magnitude smaller than $\Delta_{V}$, and thus we only minimize $\Delta_{V}$ over $\Gamma$ and $b_0/b_o$. $b_o$ is determined by matching the droplet’s terminal speed far from wall in the DP simulations and experiments. In particular, we equate the dimensionless terminal speed,

$$V_t \equiv \frac{V_g}{V_{\text{exp}}} = \frac{mg}{b_o \sqrt{\gamma_{\text{exp}}}}, \quad (15)$$

in the experiments and DP simulations.

In Fig. 2(a) and (b), we show the droplet shapes as a function of distance $h$ from the orifice (with width $w = 0.4\sigma$) from trajectories of gravity-driven droplet flows through narrow channels in experiments and DP simulations using the values $\Gamma^* = 0.16 \pm 0.01$ and $b_0^*/b_o = 0.064 \pm 0.003$ that minimize $\Delta_{V}$. The error bars for $\Gamma^*$ and $b_0^*$ are the standard deviations in $\Gamma^*$ and $b_0^*$ from fitting the the DP simulations to eight independent experimental trials. For the experiments in Fig. 2(a), the terminal velocity $V_t \approx 7 \text{ mm/s}$ far from the wall, undeformed droplet diameter $\sigma \approx 3.5 \text{ mm}$, and tilt angle $\theta \approx 28^\circ$. Thus, for the data in Fig. 2(a), we estimate the dimensionless surface tension for the droplets to be $\Gamma_{\exp} \approx 0.57$ and the dimensionless terminal velocity to be $V_t \approx 0.08$. In Fig. 2(c) and (d), we plot $\Delta_{\text{sh}}$ and $V_g$ as a function of $h/\sigma$ for both the calibrated DP simulations and experiments for the data presented in Fig. 2(a) and (b). We find that the droplet shapes and speed profiles for the DP simulations and experiments are similar over the full range of $h$, with $\Delta_{\text{sh}} \approx 0.01$ and $\Delta_{V} \approx 0.09$. Note that despite the simplicity of the 2D DP simulations, $\Gamma^*$ is comparable to $\Gamma_{\exp}$ (with $\Gamma^*/\Gamma_{\exp} \approx 0.3$), emphasizing the predictive power of 2D DP simulations.

In Fig. 4 we compare the results for the calibrated DP simulations and experiments of gravity-driven droplet flows in nar-
row channels for two additional orifice widths and tilt angles. In Fig. 3 (a) and (b), we show $\alpha$ and $v_\eta$ versus $h/\sigma$ for $w/\sigma = 0.90$, $\theta \approx 8.5^\circ$, $\sigma \approx 1.98$ mm, $v_\eta \approx 0.97$ mm/s, $V_j \approx 0.028$, and $\Gamma_{\exp} \approx 5.80$. In Fig. 3 (c) and (d), we show $\alpha$ and $v_\eta$ versus $h/\sigma$ for $w/\sigma = 0.70$, $\theta \approx 5.0^\circ$, $\sigma \approx 3.30$ mm, $v_\eta \approx 1.1$ mm/s, $V_j \approx 0.032$, and $\Gamma_{\exp} \approx 3.47$. We can obtain the dimensionless line tension $\Gamma^{**}$ that minimizes $\Delta_v$ for the experimental data in Fig. 3 (with tilt angle $\theta^{**}$ and undeformed droplet diameter $\sigma^{**}$) using the values of $\Gamma^{**}$, $\theta^{**}$, and $\sigma^{**}$ from the calibrated experiments and DP simulations in Fig. 2 using $\Gamma^{**}/\Gamma^{*} = (\sigma^{**}/\sigma^{*})^2 \sin \theta^{**}/(\sigma^{**}/\sigma^{*})^2 \sin \theta^{**}$. We find $\Gamma^{**} = 1.7 \pm 0.1$ for $\theta^{**} = 8.5^\circ$ and $\sigma^{**} \approx 1.98$ mm (with $\Gamma_{\exp} \approx 5.80$ and $\Gamma^{**} = 1.08 \pm 0.09$ for $\theta^{**} = 5.0^\circ$ and $\sigma^{**} = 3.30$ mm (with $\Gamma_{\exp} \approx 3.47$). For all tilt angles and droplet diameters studied, we find that the ratio of the dimensionless line tension that minimizes $\Delta_v$ to the dimensionless surface tension in experiments satisfies $\Gamma^{**}/\Gamma_{\exp} \approx 0.3$, which likely stems from the fact that we compare values of the dimensionless line tension in 2D to values of the dimensionless surface tension in 3D. We find that the near-wall drag coefficients that minimize $\Delta_v$ do not depend strongly on droplet diameter and tilt angle, $b^*_h/b_\eta \approx 0.06-0.2$.

A key feature of the droplet speed profile $v_\eta$ is that it can possess nonmonotonic behavior and $v_\eta$ can even exceed the terminal speed $v_j$ as the droplet exits the orifice. For example, in Fig. 3 (d), $v_\eta$ overshoots $v_j$ near $h/\sigma \approx 0.3$. Similar phenomena have been observed in pressure-driven droplet flows in obstacle arrays, where the pressure drop across the orifice changes from positive to negative, which indicates that the droplet speed exceeds the background fluid velocity. To further investigate the effects of the gravitational driving on the nonmonotonic speed profile, we show $v_\eta$ versus $h/\sigma$ for fixed $w/\sigma = 0.7$ for several values of the tilt angle: $\theta = 2.7^\circ$, $4.5^\circ$, and $6^\circ$ in Fig. 4. The nonmonotonic droplet speed profile can be understood by analyzing energy transfer while the droplet moves through the narrow channel. The droplet first impacts the chamber wall at $h/\sigma < 0$ and deforms while squeezing through narrow orifice. During this process, the kinetic and gravitational energy of the droplet transfers into surface energy of the deformed droplet, which corresponds to the first speed minimum near $h/\sigma \approx -0.25$. The large surface energy is transferred into kinetic energy as the droplet continues to move through the orifice, which can give rise to speeds that exceed the terminal velocity near $h/\sigma \approx 0.3$. However, the excess kinetic energy is quickly dissipated by viscous drag, and $v_\eta$ decreases again, reaching a second local minimum near $h/\sigma \approx 0.5$. Finally, the speed increases again, approaching $v_j$ as the droplet moves far from the orifice since the drag coefficient $\zeta$ decreases toward $b_\eta$ for $h/\sigma \gg 1$. (See Eq. 6)

As shown in Fig. 4, the amplitude of the overshoot in the droplet speed increases with decreasing $g$ and $\theta$. The excess shape energy $\Delta u_e$ gained during droplet deformation is largely determined by the width of the orifice, $w/\sigma$. In contrast, the droplet kinetic energy at the terminal speed $E_t$ decreases with increasing $g$. Thus, for $h > 0$, the excess surface energy $\Delta u_s$ transferred into kinetic energy $E_t \sim \Delta u_s$, generates enhanced droplet speed overshoots $v_\eta \sim \sqrt{E_t/E_j}$ as $g$ or $\theta$ decreases.

4.2 Clogging of single droplets in obstacle arrays

Droplet shape dynamics plays an important role in determining the motion and clogging of droplets flowing through obstacle arrays. For example, in Fig. 5 (a) and (c), we show that droplets can wrap around obstacles and squeeze through narrow constrictions formed by adjacent obstacles. These shape change mechanisms do not occur for nearly rigid particles and can be accurately modeled using the calibrated DP simulations with surface tension described in Sec. 4.1 as shown in Fig. 5 (b) and (d).

For minimum obstacle separation $w_{ob}/\sigma \gg 1$, single droplets flow continuously through obstacle arrays without clogging. However, when $w_{ob}/\sigma \ll 1$, single droplets can become permanently clogged within the obstacle array. We assume that as the droplet moves through the obstacle array, it successively interacts with each obstacle, with each interaction contributing to the probability of clogging. This scenario suggests that particle clogging is a Poisson process, confirmed by numerous studies prior studies. This means that the droplet can move a distance $r > 0$ without clogging with cumulative probability distribution.
function,
\[ P(r) = e^{-r/\lambda}, \tag{16} \]
where \( \lambda \) is clogging decay length such that \( P(\lambda) = e^{-1} \). Additional details concerning the measurements of \( P(r) \) in the DP simulations are included in Appendix C.

Several previous studies of clogging of soft particles in hoppers have shown that softer particles (i.e. with smaller elastic moduli) are less likely to form clogs\[36,38\], and thus would possess larger values of the clogging decay length \( \lambda \). Does this result also hold for the highly deformable droplets considered in this study? To address this question, we carry out numerical simulations of single droplets (using the SP and DP models) moving through obstacle arrays in the clogging regime with \( w_{ob}/\sigma \leq 0.5 \). We measure \( \lambda \) as a function of \( \Gamma \) (for the DP model), \( K_w \) (for the SP model), and the deformability \( \delta/\sigma \), given by the fraction of the droplet diameter that deforms under its own weight. We find that the wrapping mechanism plays an important role in determining the clogging statistics for single droplets moving through obstacle arrays. In particular, clogs can form when droplets wrap around an obstacle as shown in Fig. 6(a). The droplet is mainly supported by the obstacle around which the droplet has wrapped, with additional stabilizing forces from adjacent obstacles. For less deformable particles, clogs form when droplets cannot squeeze through narrow channels between adjacent obstacles as shown in Fig. 6(b).

In Fig. 6(c), we plot \( \lambda/\sigma \) as a function of the line tension \( \Gamma \) from DP model simulations of single droplets moving through obstacle arrays for \( \sigma_{ob}/\sigma = 0.3 \) and \( w_{ob}/\sigma = 0.2, 0.3, 0.4, \) and 0.5. We find that \( \lambda/\sigma \) versus \( \Gamma \) is non-monotonic. At large \( \Gamma \), where droplet shape deformation is small, decreasing \( \Gamma \) increases \( \lambda \), which implies that softer particles are less likely to clog. However, at small \( \Gamma \), where droplet shape deformation is large, we find the opposite behavior. In this regime, decreasing \( \Gamma \) decreases \( \lambda \), which implies that softer particles are more likely to clog. The non-monotonic behavior of \( \lambda(\Gamma) \) is caused by the two different mechanisms for clogging (wrapping versus squeezing) as shown in Fig. 6(a) and (b). At large \( \Gamma \), droplets become clogged when squeezing between obstacles. It is easier for nearly rigid droplets to squeeze through the gaps between obstacles as \( \Gamma \) decreases, which gives rise to decreased clogging probability and increased \( \lambda \). In contrast, at small \( \Gamma \), highly deformable droplets can easily squeeze through narrow gaps between obstacles, but they can wrap around an obstacle and clog. Thus, increased deformability enhances the wrapping mechanism, increasing the clogging probability, and decreasing \( \lambda \). In Fig. 6(c), we also show that \( \lambda \) increases with \( w_{ob} \) as expected, since wider gaps between obstacles reduces clogging.

We also carry out SP simulations of single droplets moving through the same obstacle arrays in Fig. 6 to understand the effects of droplet stiffness versus deformability on clogging. In Fig. 7(a), we plot \( \lambda/\sigma \) versus the dimensionless stiffness \( E_w \) for the SP model; \( \lambda/\sigma \) decreases monotonically with increasing \( E_w \) over the full range of \( E_w \), and as expected, \( \lambda/\sigma \) increases with \( w_{ob}/\sigma \). These results emphasize that the SP model can only generate clogs via the squeezing mechanism, not the wrapping mechanism.

To directly compare the results for single droplet motion through obstacle arrays using the SP and DP models, we consider the droplet deformation \( \delta \) caused by its own weight\[36\]. (See Fig. 7(b).) To calculate \( \delta \), we initially turn off gravity and place an SP droplet or a DP droplet with diameter \( \sigma \) next to a wall. We then turn on gravity and measure the distance \( \delta \) that the droplet sinks. In Fig. 7(c), we plot \( \lambda/\sigma \) versus \( \delta/\sigma \) for both the SP and DP models of single droplet motion through obstacle arrays. \( \lambda/\sigma \) versus \( \delta \) is similar for the SP and DP models for \( \delta/\sigma < 0.25 \), but the results differ significantly for \( \delta/\sigma > 0.25 \). At large \( \delta/\sigma \), overlaps between the obstacles and SP droplets are not strongly penalized, clogs are highly unlikely, and \( \lambda/\sigma \rightarrow \infty \). However, at large \( \delta/\sigma \), DP droplets are extremely deformable, can wrap around obstacles, and thus \( \lambda/\sigma \rightarrow 0 \). Comparing the results for the SP and DP models allows us to assess the importance of shape deformability, particle softness, and volume conservation in determining the clogging probability.

### 4.3 Continuous single-droplet flows in obstacle arrays

We now investigate continuous flows of single droplets through obstacle arrays (using the SP and DP models). We calculate the average droplet speed \( \langle v_g \rangle \) (over each droplet trajectory and multiple random obstacle arrays, see Appendix D) as a function of the size and placement of the obstacles. For simplicity, we set \( \nu_i = 0.03 \) and \( b_0 = 0 \), but the results also hold for non-zero \( b_0 \). (See Appendix E.) Intuitively, one might expect that increasing
the line tension $\Gamma$ would lead to increased resistance to droplet shape changes, and thus decreases in $\langle v_g \rangle$. However, in Fig. 8(a), we find that $\langle v_g \rangle$ increases with $\Gamma$ for $w_{ob}/\sigma \geq 1$. The decrease in $\langle v_g \rangle$ with decreasing $\Gamma$ occurs because droplets with small surface tension can easily wrap around an obstacle. When droplets wrap around an obstacle, the two ends of the droplet extend in the flow direction until the droplet surface energy matches the work done by the driving force. After the ends of droplet stop moving in the flow direction, one end of the droplet moves in the opposite direction of the driving force, which allows the droplet to unwrap. This wrapping and unwrapping process significantly slows droplet movement and decreases $\langle v_g \rangle$.

In Fig. 8(b), we quantify continuous flows after fixing $w_{ob}/\sigma = 1.0$ and varying $\sigma_{ob}/\sigma$. We plot $\langle v_g \rangle/\nu$ versus $\Gamma$ for $\sigma_{ob}/\sigma = 0.3$, 0.6, and 0.9. We find that increasing $\sigma_{ob}/\sigma$ increases $\langle v_g \rangle$ at small $\Gamma$, indicating that it is more difficult for droplets to wrap around larger obstacles. In the large-$\Gamma$ limit, the single-droplet flows for all $\sigma_{ob}/\sigma$ approach the same $w_{ob}$-dependent plateau value in $\langle v_g \rangle$. Droplet wrapping does not occur in this regime since droplet deformation would require large surface energy.

In Fig. 9(a), we show $\langle v_g \rangle/\nu$ versus the droplet-obstacle stiffness $E_w$ using the same obstacle arrays in Fig. 8 for the SP model. In contrast to the DP simulations, we find that $\langle v_g \rangle/\nu$ decreases with increasing $E_w$, which indicates that softer particles flow more rapidly for the SP model. This result stems from the fact that
droplet softness in the SP model is represented by larger allowed overlaps between the droplet and obstacles. Therefore, in the \( E_w \to 0 \) limit, droplets would flow through obstacles without slowing down, resulting in \( \langle v_o \rangle / v_t \to 1 \). In contrast, in the \( \Gamma \to 0 \) limit in DP simulations, the droplets wrap around each obstacle that they encounter. This difference in the flow behavior of single SP and DP droplets emphasizes the importance of modeling explicit deformability in droplet flows through obstacle arrays. In Fig. 9 (b), we plot \( \langle v_o \rangle / v_t \) versus droplet deformation \( \delta_s / \sigma \) at \( w_{ob} / \sigma = 1.1 \) for both the DP and SP models. We find that \( \langle v_o \rangle / v_t \) for the SP and DP models converge to same value in the rigid-droplet limit \( (\delta_s / \sigma \to 0) \), but they possess diverging behavior for large droplet deformation, \( \delta_s / \sigma \to 1 \).

![Graph](image)

Fig. 9 (a) Average speed of the center mass of the droplet in the direction of gravity \( \langle v_o \rangle / v_t \) plotted as a function of \( E_w \) for SP simulations of continuous single-droplet flows through obstacle arrays with minimum gap sizes \( w_{ob} / \sigma = 1.0 \) (red), 1.1 (green), 1.2 (blue), and 1.3 (black). The arrow indicates increasing \( w_{ob} / \sigma \). (b) \( \langle v_o \rangle / v_t \) plotted as a function of \( \delta_s / \sigma \) for single-droplet flows through obstacle arrays with \( w_{ob} / \sigma = 1.1 \) for both SP (black) and DP (red) models.

5 Discussion and Conclusions

In this article, we carried out coordinated experiments and numerical simulations of gravity-driven flows of single droplets in narrow channels and obstacle arrays. We first developed deformable particle (DP) simulations with line tension in 2D and calibrated them to the experiments. The DP simulations recapitulate the experimental results for the droplet shape and speed for flows within narrow channels with an accuracy of less than 10%. Using the calibrated DP simulations, we find several important results concerning single-droplet flows in narrow channels and obstacle arrays. First, the droplet speed possesses nonmonotonic behavior as the droplet traverses the narrow channels. The droplet speed can even overshoot the terminal speed and the overshoot is enhanced as the gravitational driving is decreased. Second, we showed that the clogging probability is nonmonotonic with the line tension for droplets flowing through obstacle arrays. The nonmonotonic behavior is caused by two distinct clogging mechanisms (droplet wrapping and squeezing) in obstacle arrays. Stiffer droplets become trapped when they cannot squeeze in between obstacles, while highly deformable droplets clog when they wrap around obstacles, resulting in high clogging probabilities for droplets with both large and small deformabilities. We also compared the DP simulation results for flows in obstacle arrays to those for the frequently used soft particle (SP) model, which does not include explicit shape deformability. The DP and SP simulations of single-droplet flows in obstacle arrays are similar in the rigid-droplet limit, but diverge for deformable droplets since the SP model can only model clogging via the squeezing mechanism (and not the wrapping mechanism).

These results suggest several promising future research directions. First, the current numerical simulations employ the DP model with line tension in 2D. When calibrating the 2D DP model to the experimental results, we obtain values for the dimensionless line tension \( \Gamma^* \) that are a factor of \( \sim 3 \) smaller than the dimensionless surface tension \( \Gamma_{exp} \) in experiments. Thus, in future work we will develop the 3D DP model \(^{32}\) with surface tension to describe quasi-2D flows of droplets in narrow channels and obstacle arrays. Second, in the current work, we do not quantitatively model the background fluid. However, the hydrodynamics of the background fluid can influence the droplet dynamics, such as causing the droplets to follow stream lines, which can prevent droplet-obstacle contact. Therefore, in the future, we can include the effects of Stokesian dynamics of the background fluid on the droplet motion.

Third, we observe in experiments that droplets undergoing sufficiently large deformations break up into smaller droplets. However, our current numerical simulations do not account for this phenomenon. In future studies, we will develop a quantitative framework to predict the conditions under which droplet breakup occurs and use it to determine the size distribution of daughter droplets emerging from obstacle arrays. This improved understanding of droplet breakup will be valuable in microfluidic applications \(^{35,56}\), where generating droplets of a specific size is required \(^{22}\). We also plan to investigate multi-droplet flows through obstacle arrays. The efficiency of the DP model will enable us to simulate systems consisting of thousands of droplets, and explore under what conditions droplet interactions give rise to coalescence, where two droplets in contact merge to form a larger droplet \(^{35}\). Droplet breakup and coalescence play competing roles in determining the equilibrium size distribution of droplets moving through obstacle arrays \(^{35}\). In future studies, we will investi...
gate their combined effects to quantitatively predict the droplet size distribution exiting from the obstacle arrays.

\[ \frac{v_g}{v_t - v_g} = b_0 \left( 1 + \frac{b_0}{b_\infty} \frac{\sigma}{d} \right)^\alpha \approx -Mg = v_t b_\infty. \] (17)

After rearranging Eq. 17 we find
\[ b_0 \left( \frac{\sigma}{d} \right)^\alpha \approx \left( \frac{v_t}{v_g} - 1 \right) b_\infty. \] (18)
\[ \alpha \log_{10} \left( \frac{d}{\sigma} \right) \approx \log_{10} \left( \frac{v_t}{v_g} - 1 \right) + \log_{10} \left( \frac{b_\infty}{b_0} \right). \] (19)

In Fig. 10 we plot \( v_g/(v_t - v_g) \) versus \( d/\sigma \) on a \( \log_{10}/\log_{10} \) scale, where \( d \) is the minimum distance between chamber wall and the droplet. The droplet shown in the inset has diameter \( \sigma = 0.98 \text{ mm} \), tilt angle \( \theta = 5^\circ \), and terminal velocity far from wall \( v_t = 0.65 \text{ mm/s} \). We find that the experimental data in Fig. 10 is well-fit by a power-law exponent \( \alpha = 4/3 \).

Appendix B

In this Appendix, we investigate the effects of the number of vertices \( N_v \) and perimeter elasticity \( K_I \) on the results for droplet flows through narrow channels using the DP simulations. The DP model with \( N_v \to \infty \) and \( K_I \to 0 \) is the most accurate for describing droplet flows. However, to limit computational time, we choose \( N_v \) such that it is sufficiently large that further increases do not significantly change the results. In Fig. 11 we show the droplet speed profile, \( v_g/v_t \), versus \( h/\sigma \) for \( N_v = 32, 64, 128, 256, 512, \) and 1024.

We find that for \( N_v \leq 64 \), the velocity profile is quite noisy. For \( N_v \geq 128 \), the velocity profile is smooth on a linear scale. For the DP simulations presented here, we use \( N_v = 256 \).

Appendix C

In this Appendix, we include additional details concerning the definition of a permanent clog and the clogging decay length \( \lambda \) for droplets moving in obstacle arrays. In the simulations, a clog occurs when the droplet kinetic energy \( E_k \) (normalized by the kinetic energy at the terminal speed) decays below a threshold \( E_k/E_t < 10^{-17} \). In Fig. 13 (a), we show examples of \( E_k/E_t \) versus time \( t/t_0 \) for cases when the droplet clogs (\( w_{ab}/\sigma = 0.3 \)) and does not clog (\( w_{ab}/\sigma = 1.1 \)).

To determine the clogging decay length \( \lambda \), we first measure the cumulative probability \( P \) that the droplet does not clog as a function of the droplet displacement \( r \) in the obstacle array. We run \( N_v = 50 \) simulations for a total time \( T = 2 \times 10^5 t_0 \) with dif-
In Fig. 15, we plot $\langle v_g \rangle$ and SP simulations, both have the same form for the drag force. As discussed previously, the SP model does not include distance-dependent drag forces (Eq. 6). In Fig. 9 (c) and Fig. 7 (b), we set $w_{ob}/\sigma = 0.2$. At the beginning of each simulation, $r = 0$ and $P = 1$. As each droplet traverses the obstacle array, the cumulative probability $P$ of not clogging decreases exponentially with increasing $r$.

Appendix D

In this Appendix, we show that the DP simulations of single droplet flows in obstacle arrays can reach a continuous flow regime (with no permanent clogs) for minimum obstacle spacing $w_{ob}/\sigma > 1$. In Fig. 14, we plot the droplet displacement $r$ versus time $t$ for DP simulations with $\Gamma = 0.041$. We determine the steady state speed $v_t$ for a single-droplet trajectory in a given obstacle array by measuring the slope of $r/\Gamma$ versus $t/t_0$. $\langle v_g \rangle$ is averaged over 10 random obstacle arrays.

Appendix E

In this Appendix, we show that the choice of the near-wall drag coefficient $b_0$ does not qualitatively affect the results in Sec. 4.3. As discussed previously, the SP model does not include distance-dependent drag forces (Eq. 6). In Fig. 9 (c) and Fig. 7 (b), we set $b_0 = 0$ in the DP simulations so that when we compare the DP and SP simulations, both have the same form for the drag force. In Fig. 15, we plot $\langle v_g \rangle/\sqrt{t}$ versus $\Gamma$ for single droplets flowing through obstacle arrays with $w_{ob}/\sigma = 1.0$. All other parameters are the same as those in Fig. 8(a), except the near-wall drag coefficient varies: $b_0/b_{sw} = 0, 0.2,$ and 0.4. We find that increasing $b_0/b_{sw}$ decreases $\langle v_g \rangle/\sqrt{t}$, but it monotonically increases with $\Gamma$ for all $b_0/b_{sw}$.

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of the dimensionless line tension

Fig. 14 Droplet displacement $r$ in the direction of gravity $r/\sigma$ plotted as a function of time $t/t_0$ as the droplet traverses an obstacle array with $w_{ob}/\sigma = 1.1$ and $\sigma_{ob}/\sigma = 0.3$. The arrow indicates increasing $\Gamma$ from 0.04 (blue) to 1 (red).

Fig. 15 Average speed of the center mass of the droplet in the direction of gravity $\langle v_y \rangle$ normalized by the terminal velocity $v_t$ plotted as a function of the dimensionless line tension $\Gamma$ for obstacle arrays with $w_{ob}/\sigma = 1.0$. All other parameters are the same as those used in Fig.14 (a), except we vary the near-wall drag coefficient: $b_0/b_\infty = 0$ (red), 0.2 (blue), and 0.4 (black). The arrow indicates increasing $b_0/b_\infty$.


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